

CHM 3411 – Problem Set 1

Due date: Wednesday, January 16th

Do all of the following problems. Show your work.

1) The Planck distribution law is

$$M(\lambda, T) d\lambda = \frac{2\pi hc^2}{\lambda^5 [\exp(hc/\lambda kT) - 1]} d\lambda \quad (1.1)$$

a) Show that in the limit $\lambda \rightarrow \infty$ eq 1.1 reduces to the classical (Rayleigh-Jeans) expression for blackbody radiation

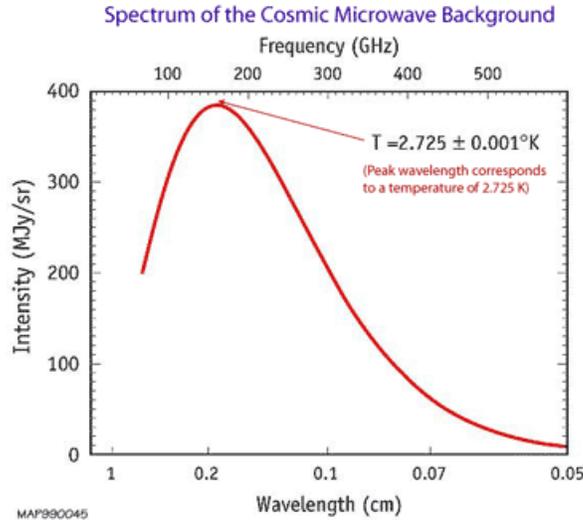
$$M_{RJ}(\lambda, T) d\lambda = \frac{2\pi ckT}{\lambda^4} d\lambda \quad (1.2)$$

b) Starting with eq 1.1, derive Wien's law

$$\lambda_{max} T = A_W = \text{constant} \quad (1.3)$$

and find the value predicted for A_W , the Wien's law constant (Note: If you do this problem correctly you will get a final expression for A_W that cannot be solved exactly, but can be used to find the value for A_W either by making a reasonable approximation or by numerical methods.)

2) The cosmic microwave spectrum believed to be the residual radiation from the "big bang" (as obtained by the COBE or COsmic Background Explorer satellite) is given below



a) Assuming that the background radiation is described by the Planck distribution law, find the location of the peak wavelength for the observed radiation. Give your answer in units of millimeters (mm). In what region of the electromagnetic spectrum does the peak occur?

b) Find the value for $\xi(T)$, the energy density of a blackbody at a temperature equal to that of the cosmic background radiation. (Recall that $M(T) = (c/4) \xi(T)$). Give your answer in units of J/m^3 .

3) In 1897, Wilhelm Wien derived a formula for blackbody radiation (using thermodynamic arguments) that for a time was believed to be correct. His formula, the Wien distribution law, is

$$M_W(\lambda, T) d\lambda = \frac{2\pi hc^2}{\lambda^5} \exp(-hc/\lambda kT) d\lambda \quad (3.1)$$

a) Starting with eq 3.1, find the expression for $M_W(\nu, T) d\nu$ (Hint: You will have to substitute both for λ and for $d\lambda$.)

b) Using your answer to part a, find an expression for the total intensity of light predicted to be emitted by a blackbody according to Wien's distribution law.

$$M_W(T) = \int_0^\infty M_W(\nu, T) d\nu \quad (3.2)$$

Does your result agree with the Wien distribution law ($M(T) = \sigma T^4$)? If so, find the value predicted for σ (in units of $W/m^2 \cdot K^4$, recalling that $W = \text{watt} = J/s$). (Note: In principle we could find $M_W(T)$ by integrating eq 3.1, but in practice it is easier to do the integration over ν rather than over λ .)

4) Experimental data from a study of the photoelectric effect for potassium metal are given below. Based on the data, find the following:

- λ_0 , the critical wavelength for potassium (in units of nm).
- Φ , the value for the work function for potassium metal (in units of eV).
- h , the experimental value for the Planck constant obtained from the data (in units of $J \cdot s$).

Wavelength (nm)	E_{Kmax} (eV)	Wavelength (nm)	E_{Kmax} (eV)
350.	1.44	509.	0.30
385.	1.09	553.	0.08
455.	0.56	608.	n/a

5) The observation by Dulong and Petit (1819) that the molar heat capacity of solid metals at room temperature is approximately $C = 3R = 24.9 J/mol \cdot K$ was useful in finding the correct value for atomic mass for a number of different metallic elements, as shown in this problem.

a) 76.47 g of a sample of pure metal (M) will completely react with 11.81 g of oxygen gas (O_2) to form a pure metal oxide. Find the atomic mass of the metal assuming the following for the chemical formula of the metal oxide – MO, MO_2 , M_2O , M_2O_3 .

b) The specific heat of the metal, measured at room temperature, was found to be $s = 0.128 J/g \cdot K$. Based on this value for s , the Dulong-Petit law, and your answer to part a of this problem, find the atomic mass of the metal.

6) A typical wavelength used in X-ray crystallography is $\lambda = 0.100 \text{ nm}$ (approximately the size of the spacing between atoms in a crystal).

a) Find the velocity of a beam of monoenergetic electrons and a beam of monoenergetic neutrons that has a de Broglie wavelength $\lambda_{dB} = 0.100 \text{ nm}$.

b) It is usually easier to generate a beam of monoenergetic electrons than it is a beam of monoenergetic neutrons. Suggest a reason why this is true.