CHM 6480 – Problem Set 1
Due date: Thursday, September 3rd (by noon)
Do all of the following problems. Show your work.

1) The Wien distribution law for the intensity of light emitted by an ideal blackbody

$$M(\lambda,T) = \frac{a}{\lambda^5} \exp(-b/\lambda T) \qquad (1.1)$$

was an early attempt to model experimental data for blackbody radiation. It reproduced a number of features in the experimental data, but was eventually found to be incorrect.

a) Starting with eq 1.1, find an expression for λ_{max}, the wavelength of maximum intensity of emitted light for an ideal blackbody. Give your final answer in terms of a, b, and T.

b) Experimentally, the wavelength of maximum intensity of emitted light for an ideal blackbody is given by the expression

$$\lambda_{max} T = A_W \qquad (1.2)$$

where $A_W = 2.898 \times 10^6$ nm•K.

Is your answer in a consistent with the above experimental result? If your answer is yes, find the value for A_W in terms of a and b.

c) The total intensity of emitted light by an ideal blackbody is given by the expression

$$M(T) = \int_0^\infty M(\lambda,T)\, d\lambda \qquad (1.3)$$

Find the expression for M(T) predicted from eq 1.1.

d) Experimentally, the total intensity of emitted light for an ideal blackbody is given by the expression

$$M(T) = \sigma T^4 \qquad (1.4)$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^2•K^4 (where W = watts, 1 watt = 1 J/s).

Is your answer in c consistent with the above experimental result? If your answer is yes, find the value for σ in terms of a and b.

e) Starting with eq 1.1, find the expression for $M(\lambda,T)$ in the limit $\lambda \to \infty$.

f) The classical (Rayleigh-Jeans) expression for the intensity of emitted light for an ideal blackbody is given by the expression

$$M(\lambda,T) = \frac{2\pi ckT}{\lambda^4} \qquad (1.5)$$

where $k = 1.381 \times 10^{-23}$ J/K is the Boltzmann constant. While incorrect at small values of wavelength, eq 1.5 agree with experiment in the limit $\lambda \to \infty$.

Is your answer in e consistent with the Rayleigh-Jeans equation in the limit $\lambda \to \infty$? If your answer is yes, find a relationship between the Wien's law constants a and b and the constants appearing in eq 1.5.

2) A particular metal is studied in a photoelectric effect experiment. When the wavelength of light used to illuminate the metal is λ = 254. nm, the maximum kinetic energy for the electrons that are emitted is $E_{K,max}$ = 2.74 eV (1 eV = 1.602×10^{-19} J). Based on this information find the following:

a) Φ_0, the work function for the metal. Give your final answer in units of eV.

b) λ_0, the critical wavelength for the metal. Give your final answer in units of nm.

3) Crystals can be studied by both electron diffraction and neutron diffraction. Each technique gives useful information about the crystal structure of a solid.

The spacing between adjacent atoms in a copper crystal is d = 0.228 nm.

a) Find the velocity (in units of m/s) for a beam of electrons whose de Broglie wavelength is λ_{dB} = 0.220 nm. (m_e = 9.109 x 10^{-31} kg).

b) Find the velocity (in units of m/s) for a beam of neutrons whose de Broglie wavelength is λ_{dB} = 0.220 nm. (m_n = 1.675 x 10^{-27} kg).

4) There are a number of proofs in quantum mechanics that are both useful and surprisingly easy. Here we explore one of these.

Let f(x) be a "well behaved" function whose domain is the same as a particular TISE. Because the functions ψ_n that are solutions to a TISE form a complete orthonormal set of functions (basis functions), we can say:

$$f(x) = c_1\, \psi_1(x) + c_2\, \psi_2(x) + c_3\, \psi_3(x) + \ldots$$

$$= \Sigma_{n=1}^{\infty}\, c_n\, \psi_n(x) \qquad (4.1)$$

a) Left multiply the left and right sides of eq 4.1 by ψ_j^*, the complex conjugate of the j^{th} solution to the TISE.

b) Integrate all of the terms from part a over the region $-\infty < x < \infty$.

c) Using the orthonormality condition, find an expression for c_j, the coefficient for the j^{th} term in eq 4.1.

d) The result in part c takes on a particularly meaningful form if f(x) is a normalized function, that is, if

$$\int_{-\infty}^{\infty} f(x)^*\, f(x)\, dx = 1 \qquad (4.2)$$

Using your result from part c, show that if f(x) is a normalized function, it follows that

$$|c_1|^2 + |c_2|^2 + |c_3|^2 + \ldots = \Sigma_{n=1}^{\infty}\, |c_n|^2 = 1 \qquad (4.3)$$

where

$$|c_n|^2 = c_n^*\, c_n \qquad (4.4)$$

(Hint: Use the normalization condition (eq 4.2) as a starting point.)

The significance of this result is that if f(x) is the wavefunction for a state in a system, and if f(x) is a normalized function, then $|c_n|^2$ represents the probability of being in the ψ_n^{th} eigenstate of the system.